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Ultrasound scattering and the study of vortex correlations in disordered flows

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In an idealized way, some turbulent flows can be pictured by assemblies of many vortices characterized by a set of particle distribution functions. Ultrasound provides a useful, nonintrusive, tool to study the spatial structure of vorticity in flows. This is analogous to the use of elastic neutron scattering to determine liquid structure. We express the dispersion relation, as well as the scattering cross section, of sound waves propagating in a ''liquid'' of identical vortices as a function of vortex pair correlation functions. In two dimensions, formal analogies with ionic liquids are pointed out.

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I. INTRODUCTION

Neutron scattering has been an essential experimental tool for the study of liquid structure for the past 30 years [1]. The fact that, in a first Born approximation, the scattering cross section is directly proportional to the static structure factor has been a source both of data and inspiration for the elaboration and verification of current theories of liquid structure [2].

At first sight, a turbulent fluid bears no relation to the microscopic structure of a liquid. However, a turbulent fluid has vortices, and these objects couple to acoustic waves in much the same way as neutrons, or electromagnetic waves, couple to matter. The purpose of the present paper is to point out a number of theoretical developments that make this analogy more precise, particularly with respect to ionic liquids.

It is well known experimentally [3–5] as well as numerically [6–10] that coherent vortical structures easily appear in turbulent flows. Three-dimensional turbulent flows of high Reynolds number exhibit many intense, long-lived and tube-like vorticity regions of fairly well defined thickness and length [10]. These filaments have a tendency to organize parallel one to each other in clusters. Although they generally represent a small part of the motion of the fluid, the filaments probably contain information on the statistics of the whole background. In a quite different context, numerical studies of two-dimensional decaying turbulence also reveal the emergence of coherent vortices with particlelike character. These vortices dominate the long-time motion of the fluid. Their size and number density are well described by scaling laws, and their dynamics are similar to the Hamiltonian motion of few point vortices [11–13]. According to these observations, two-dimensional decaying turbulence can be approximately described with a finite number of degrees of freedom. Following this approach, statistical mechanical theories of the Euler's equation in a bounded domain have been developed for systems of point vortices [14–16], and extended afterwards to continuous distributions of vorticity [17,18]. These theories where able to explain the results of two-dimensional Hamiltonian simulations [19], showing that point vortices of same circulation sign are not randomly mixed, but have a tendency to organize in a nonuniform way within the domain. In infinite domains, homogeneous two-dimensional turbulence appears to be well described by many large coherent vortices that must be spatially correlated. Quantitative experimental or numerical studies of these correlations are few, although two-dimensional turbulence has become in recent years the object of controlled laboratory experiments [20,21]

Ultrasound scattering provides a powerful, nonintrusive tool to study vortical structure in flows. The characterization of the scattered pressure of an ultrasonic plane wave at sufficiently large distances from a vorticity distribution has motivated various theoretical studies [22–27]. In a way similar to elastic neutron or x-ray scattering in liquids [1], the scattering cross section of sound waves can be related within the first Born approximation to the modulus of the Fourier transform of the vorticity field. This result has been experimentally checked for regular laboratory flows [28,29], and there is increasing interest in applying this method for the study of turbulence [30–32]. In a related development, time-reversal acoustic mirrors have been used to probe vortical flows [33].

In the present article, we analyze this scattering problem from a distribution function point of view, picturing the vorticity distribution as an assembly of many undeformable vortices whose structure is characterized *a priori* by a given set of particle distribution densities. We will consider not only the scattering cross section problem, but also the dispersion relation (or effective wave number) of an acoustic wave traveling through such a medium. We consider a plane wave that propagates in an infinite statistically homogeneous flow composed of vortices with number density n. According to multiple scattering theory (see, e.g., Refs. [34–36]), such a medium can be described on average by an effective index of refraction, which generally depends on the wave frequency. Moreover, the wave number that characterizes the mean wave has an imaginary part, equal to half the total scattering cross section per unit volume; this accounts for the attenuation of the wave amplitude due to the loss of coherence during the scattering processes. In order to get as much insight as possible while keeping algebraic complications to a minimum we take all vortices with the same absolute value of circulation.

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In a recent publication [37] we derived a general dispersion relation for a sound wave propagating in a disordered flow of mean velocity zero, and this result was applied to a ‘‘gas’’ of statistically independent identical vortices. In the next section, we extend the expression of the dispersion relation for a population of identical vortices of arbitrary spatial correlations. In Sec. III, we specify the study to two-dimensional systems and underline some formal analogies with ionic liquids. The results are discussed in Sec. IV, especially in connection with two-dimensional decaying turbulence. Technical details are given in the Appendix.

II. ‘‘LIQUIDS’’ OF VORTICES

The treatment of the interaction between a sound wave and a flow presented in Ref. [37] follows an usual scheme, already presented elsewhere [26]. The analysis is based on a linearization of the Euler and continuity equations for a fluid, and relies on several assumptions: the velocities associated with the acoustic wave are supposed to be much smaller than the typical velocities of the base flow, themselves much lower than the speed of sound. In this case the acoustic wave is a first order effect over the base flow, that can be considered as incompressible to zeroth order. The frequency ν_0 of the incoming wave is also supposed much higher than the frequencies associated with the flow, which is considered as frozen, and viscous effects can be neglected. The medium is then stationary and averages are made over flow configurations. In the following, we denote c_0 the speed of sound and $k_0=\nu_0/c_0$ the wave number of the acoustic wave in the flow at rest.

If the flow is composed of N undeformable vortices, its velocity field is the sum of the flow fields generated by each vortex,

$$\vec{u}(\vec{x})=\sum_{a=1}^{N} \vec{v}^{(a)}(\vec{x}). \tag{2.1}$$

The incident wave is scattered at the velocity inhomogeneities of the medium and when averages over disorder configurations are performed, the mean acoustic pressure that results from these scattering processes can be derived within the framework of multiple scattering theory [34–36,38,39]. The mean wave is the sum of coherent scattering paths and has the same direction of propagation than the incident wave, noted $\hat{k}_0$, but is described by an effective wave number $k\neq k_0$. For a weakly perturbed medium, i.e., for a flow of low Mach number $\mathcal{M}=|\vec{u}|/c_0$ (the vortex density n can be high), k can be expanded perturbatively around k_0 in multiple scattering series. Since the flow has no mean velocity, the term of order $\mathcal{M}$ is identically zero. The first nonvanishing corrections involve two diagrams of order $\mathcal{M}^2$, and the dispersion relations can be written in Fourier space as [37]

$$\begin{aligned}k^2=&k_0^2+k_0^2\frac{(\gamma-4)}{c_0^2}\frac{1}{V}\int\frac{d\vec{q}}{(2\pi)^d}\\&\times\sum_{a,b}\langle[\hat{k}_0\cdot\vec{v}^{(a)}(\vec{q})][\hat{k}_0\cdot\vec{v}^{(b)}(-\vec{q})]\rangle+k_0^2\frac{4}{c_0^2}\frac{1}{V}\\&\times\int\frac{d\vec{q}}{(2\pi)^d}\sum_{a,b}\langle[\hat{k}_0\cdot\vec{v}^{(a)}(\vec{q}-\vec{k}_0)][\hat{k}_0\cdot\vec{v}^{(b)}(\vec{k}_0-\vec{q})]\rangle\\&\times(\hat{k}_0.\vec{q})^2\mathcal{G}_{k_0}^{(0)}(\vec{q}),\end{aligned} \tag{2.2}$$

where $\gamma=c_p/c_v$ is the ratio of specific heats at constant pressure and constant volume, V the volume occupied by the flow, d the space dimension, and

$$\mathcal{G}_{k_0}^{(0)}(\vec{q})=\lim_{\eta\to 0^+}\frac{1}{q^2-(k_0+i\eta)^2} \tag{2.3}$$

is the Fourier transform of the free-space Green's function. The second term of the right-hand side of Eq. (2.2) is real and proportional to the kinetic energy of the flow per unit volume. The real part of the last term of the right-hand side is also of order of the kinetic energy per unit volume, but introduces dispersion. This last term has an imaginary part, and using the residue theorem, one deduces that $\mathrm{Im}(k)\equiv\Lambda^{-1}$ is given by

$$\begin{aligned}\Lambda^{-1}=&\frac{\sigma_T^*}{2}\\=&\frac{\pi k_0^{d+1}}{c_0^2}\frac{1}{V}\int\frac{d\Omega^{(d)}}{(2\pi)^d}\\&\times\sum_{a,b}\langle[\hat{k}_0\cdot\vec{v}^{(a)}(k_0\hat{q}-\vec{k}_0)][\hat{k}_0\cdot\vec{v}^{(b)}(\vec{k}_0-k_0\hat{q})]\rangle\\&\times(\hat{k}_0\cdot\hat{q})^2,\end{aligned} \tag{2.4}$$

where σ_T^* is the total scattering cross section per unit volume, $\hat{q}=\vec{q}/|\vec{q}|$, $d\Omega^{(d)}$ is the solid angle measure, and Λ is then the attenuation length of the coherent wave [40]. It is assumed that the attenuation per wavelength is small ($\Lambda k_0\gg 1$), a condition that is fulfilled for a wide range of wavelengths if the Mach number is small. [See Ref. [41] for a fuller discussion of the validity of Eqs. (2.2) and (2.4)].

If the vortices are all identical, the Fourier transform $\vec{v}^{(a)}(\vec{q})$ of the flow field generated by any vortex (a) has the form

$$\vec{v}^{(a)}(\vec{q})=\int d\vec{x}\,\vec{v}(\vec{x}-\vec{x}_a,\hat{a})e^{-i\vec{q}\cdot\vec{x}}, \tag{2.5}$$

where $\vec{x}_a$ is the position of the vortex and $\hat{a}$ an angle describing its global orientation; $\vec{v}$ is a fixed velocity function that depends on the vortex shape. The terms in Eq. (2.2) are averaged over all the positions and orientations $\{\vec{x}_a,\hat{a}\}$.

In Ref. [37], we have supposed that the cross terms in the sums of Eqs. (2.2) and (2.4) vanish. This situation is encountered either when the flow fields of the vortices (a) and (b) do not overlap ($\vec{v}^{(a)}\cdot\vec{v}^{(b)}\simeq 0$), i.e., if the number density $n=N/V$ is small, or if the density is high but the vortices are spatially uncorrelated ($\langle\vec{v}^{(a)}\cdot\vec{v}^{(b)}\rangle\simeq 0$, or mean-field-like assumption).

If correlations between vortices are present, the off-diagonal terms of the Born integrals of Eqs. (2.2) and (2.4) do not vanish. It is thus convenient to introduce a pair correlation function for the rigid vortices, noted g. With a normalized angle measure, this distribution function is such that

$n^2 g(\vec{x}_a,\vec{x}_b,\hat{a},\hat{b})d\vec{x}_a d\vec{x}_b d\hat{a}d\hat{b}$ represents the number of pairs per unit volume and per unit angle formed by two vortices in configurations $\{\vec{x}_a,\hat{a}\}$ and $\{\vec{x}_b,\hat{b}\}$, respectively. Consequently,

$$\langle v_i(\vec{x}-\vec{x}_a,\hat{a})v_j(\vec{x}-\vec{x}_b,\hat{b})\rangle = \frac{1}{V^2}\int d\vec{x}_a d\vec{x}_b d\hat{a}d\hat{b}g(\vec{x}_a,\vec{x}_b,\hat{a},\hat{b})v_i(\vec{x}-\vec{x}_a,\hat{a}) \times v_j(\vec{x}-\vec{x}_b,\hat{b}). \tag{2.6}$$

For homogeneous systems with vanishing correlations at large distances, $g(\vec{x}_a,\vec{x}_b,\hat{a},\hat{b})=g(\vec{x}_a-\vec{x}_b,\hat{a},\hat{b})$, and $g\to 1$ when $|\vec{x}_a-\vec{x}_b|\to\infty$. Using the fact that $\int d\vec{x}_a v_i(\vec{x}-\vec{x}_a)=0$, expression (2.6) remains unchanged if g is replaced by the total correlation function $h\equiv g-1$, which has the advantage of a well-defined Fourier transform. Using the identity (2.6), the dispersion relation (2.2) can be recast as

$$\begin{aligned}k^2=k_0^2&+\frac{(\gamma-4)k_0^2}{c_0^2}\int\frac{d\vec{q}}{(2\pi)^d}d\hat{a}d\hat{b}[\hat{k}_0\cdot\vec{v}(\vec{q},\hat{a})]\\&\times[\hat{k}_0\cdot\vec{v}(-\vec{q},\hat{b})]\{n\delta(\hat{a}-\hat{b})+n^2h(\vec{q},\hat{a},\hat{b})\}\\&+\frac{4k_0^2}{c_0^2}\int\frac{d\vec{q}}{(2\pi)^d}d\hat{a}d\hat{b}[\hat{k}_0\cdot\vec{v}(\vec{q}-\vec{k}_0,\hat{a})][\hat{k}_0\cdot\vec{v}(\vec{k}_0-\vec{q},\hat{b})]\\&\times(\hat{k}_0\cdot\vec{q})^2\mathcal{G}^{(0)}_{k_0}(\vec{q})\{n\delta(\hat{a}-\hat{b})+n^2h(\vec{q}-\vec{k}_0,\hat{a},\hat{b})\},\end{aligned}\tag{2.7}$$

where $\vec{v}(\vec{q},\hat{a})$ is the Fourier transform of the flow field of the vortex located at the origin, $\vec{v}(\vec{q},\hat{a})=\int d\vec{x}\ \vec{v}(\vec{x},\hat{a})\exp(-i\vec{q}\cdot\vec{x})$. In the same way, we get from Eq. (2.4)

$$\begin{aligned}\sigma_T^*=\frac{2\pi k_0^{d+1}}{c_0^2}&\int\frac{d\Omega^{(d)}}{(2\pi)^d}d\hat{a}d\hat{b}[\hat{k}_0\cdot\vec{v}(k_0\hat{q}-\vec{k}_0,\hat{a})]\\&\times[\hat{k}_0\cdot\vec{v}(\vec{k}_0-k_0\hat{q},\hat{b})](\hat{k}_0\cdot\hat{q})^2\\&\times\{n\delta(\hat{a}-\hat{b})+n^2h(k_0\hat{q}-\vec{k}_0,\hat{a},\hat{b})\}.\end{aligned}\tag{2.8}$$

When the vortices are uncorrelated, $h=0$, we recover the results of Ref. [37,41].

III. TWO-DIMENSIONAL CASE

Two-dimensional turbulence has been the subject of many studies because of its possible applications in meteorology and oceanography, but also because it is the most accessible dimension for computational and theoretical approaches. In the present context, the two-dimensional study of the effects of vortex correlations on sound propagation is clearly simpler because of the reduced number of degrees of freedom involved. However, we hope that it can provide at least some first answers qualitatively valid in any dimension, besides the fact that the structure of turbulence itself deeply changes with the space dimensionality. In two dimensions the vorticity $\vec{\omega}$ points in the perpendicular direction, along the z axis. We consider a system composed of axisymmetric vortices, where each vortex produces an azimuthal velocity field (see Ref. [41] for an example) around a vortex core of circulation modulus Γ. Suppose that each vortex has a probability x_+ to have a circulation $+\Gamma$, and a probability x_- to have circulation $-\Gamma(x_++x_-=1)$. The two-particle distributions are described by introducing three pair correlation functions, h_{++}, h_{--}, and h_{+-}, where the sign of the subscript refers to the respective signs of the vortex circulations. At this point, it is convenient to introduce distribution functions used in the formalism of ionic liquids [42]. Let us consider the local vortex density, namely a ''charge'' density, defined as

$$\rho^Z(\vec{r})=\sum_i z_i\rho_i(\vec{r}), \tag{3.1}$$

where the sum runs over the two species of number density $\rho_i(\vec{r})$ and charge z_i (here, the orientation ± 1). The vortex, or charge, structure factor S_{ZZ} associated with the density $\rho^Z(\vec{r})$, defined by

$$S_{ZZ}(\vec{q})=\frac{\langle\rho^Z(\vec{q})\rho^Z(-\vec{q})\rangle}{N}, \tag{3.2}$$

can be rewritten as

$$S_{ZZ}(\vec{q})=1+n[x_+^2h_{++}(\vec{q})+x_-^2h_{--}(\vec{q})-2x_+x_-h_{+-}(\vec{q})]. \tag{3.3}$$

From Eq. (2.7), we can deduce the expression for the index of refraction defined as $\mathcal{N}=c/c_0=\mathrm{Re}(k_0/k)$, where c denotes the new phase velocity. Noting that $\vec{v}(\vec{x},-\hat{z})=-\vec{v}(\vec{x},\hat{z})$, and assuming that the flow is isotropic, one gets

$$\begin{aligned}\delta\mathcal{N}=1&-\frac{n(\gamma-4)}{8\pi c_0^2}\int_0^\infty dq\,q\,v^2(q)S_{ZZ}(q)\\&-\frac{2n}{c_0^2}\mathrm{Re}\int\frac{d\vec{q}}{(2\pi)^2}|\hat{k}_0\cdot\vec{v}(|\vec{q}-\vec{k}_0|)|^2S_{ZZ}(|\vec{q}-\vec{k}_0|)\\&\times(\hat{k}_0\cdot\vec{q})^2\mathcal{G}^{(0)}_{k_0}(q).\end{aligned}\tag{3.4}$$

As a consequence of the structure of Eq. (3.4), the asymptotic results obtained in Ref. [41] for the index of refraction in the limits of short and long wavelengths, remain valid here, provided that one replaces any quadratic factor v^2 by v^2S_{ZZ}.

The total scattering cross section, in turn, is given by

$$\sigma_T^*=\frac{nk_0^3}{2\pi c_0^2}\int d\Omega^{(2)}|\hat{k}_0\cdot\vec{v}(|k_0\hat{q}-\vec{k}_0|)|^2S_{ZZ}(|k_0\hat{q}-\vec{k}_0|)\times(\hat{k}_0\cdot\hat{q})^2. \tag{3.5}$$

Using the property $\vec{q}\cdot\vec{v}(\vec{q})=0$ for incompressible fluids (a correct assumption for low Mach number base flows), as illustrated by the geometrical construction of Fig. 1, the above expression can be rewritten

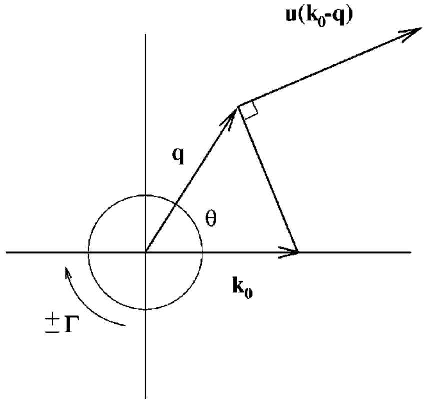

FIG. 1. Axisymmetric vortex in two dimensions.

$$\sigma_T^* = \frac{n k_0^3}{2\pi c_0^2}\int_{-\pi}^{\pi} d\theta \, \frac{\sin^2\theta \cos^2\theta}{2(1-\cos\theta)} \; v^2(k_0\sqrt{2-2\cos\theta}) \times S_{ZZ}(k_0\sqrt{2-2\cos\theta}). \tag{3.6}$$

With the same techniques presented in Ref. [41], it is easy to show that $\sigma_T^* \sim k_0^2$ when $k_0 \to \infty$. If $S_{ZZ}(q)$ behaves as q^β at small q, one gets, from Eq. (3.6),

$$\sigma_T^* \sim k_0^{5+\beta}, \quad k_0 \to 0, \tag{3.7}$$

where we have used the property $v(q\to 0) \sim q$ for an axisymmetric bounded flow.

IV. DISCUSSION

Relations (3.4) and (3.5) show that the coherent propagation of an acoustic wave through a population of identical vortices closely depends on their spatial structure. In two dimensions, the index of refraction and the attenuation length of the wave involve a circulation (or ''charge,'' by reference to ionic liquids) structure factor. Hence, if the vortices have a simple shape and generate a flow field v with well known characteristics, information on this distribution function can be deduced from the study of the acoustic properties mentioned above. Tractable analytical expressions have only been obtained in two dimensions. However, we think that they provide a qualitative understanding of the probe of vortex correlations by ultrasound techniques, even in higher space dimensions.

For its similarity with two-dimensional homogeneous turbulence [43], let us consider further the case of a neutral flow, where $x_+ = x_- = 1/2$. There are mainly two ways of considering the vortex structure of decaying two-dimensional turbulence, since there are two very different length scales in the problem. Depending on the scale of interest, one can either picture the flow as composed of many small, nearly pointlike vortices that may form large structures, or consider the flow as formed by these few large coherent vortices only. One can assume that the size of the former vortices is of order of a dissipation scale, while the size of the latter is of order of the integral scale, at which energy is initially injected.

An ideal gas of vortices is characterized by $S_{ZZ}(q)=1$; however, Eq. (3.3) shows that it is equivalent, with respect to the dispersion relation, to identical ordering between identical and opposite vortices $(h_{++}+h_{--}=2h_{+-})$. We logically check that a tendency to observe vortex pairs of identical circulation $(h_{++}+h_{--}>2h_{+-})$ would result in an increase of the kinetic energy of the flow, and hence the phase change. The limit of $S_{ZZ}(q)$ at small wave numbers is related to that of the total scattering cross section through relation (3.7) if the flow produced by one vortex is bounded in space. $S_{ZZ}(0)$ indeed represents the local fluctuations of the number of charges, and is of particular interest. A local electroneutrality assumption $[S_{ZZ}(0)=0]$ means that the ''charge'' (or circulation) of a given vortex is exactly canceled by the total ''charge'' of the vortices that surround it: at length scales of the order of a few vortex radii, the fluid has no net rotation.

Let us consider the flow at small scales, i.e., made of nearly pointlike vortices. Although not much studied in the literature, vortex pair correlations have motivated theoretical works. An exact analytical expression for the vorticity structure factor has been derived for a class of two-dimensional stationary solutions of the Navier-Stokes equation [44]. It is characterized by a Debye-Hückel-like pair distribution, $S_{ZZ}(q) \propto q^2/(q^2+k_s^2)$. Turbulence at high Reynolds number clearly exhibits quite distinct statistical properties: computer simulations rather show that vortices with same circulation sign have the tendency to organize in domains [19]. Such systems, where electroneutrality is not locally observed, are theoretically better described by other approaches, like the microcanonical formulations of the statistics of two-dimensional vortices in a bounded domain [14–18]. These theories predict that the one-point probability distribution of vortices is spatially nonuniform. To obtain this result, however, two-point correlation functions are usually neglected at first order $(S_{ZZ}=1)$.

Two-dimensional homogeneous isotropic turbulence can be conveniently described in the inertial range by uniform mixture of long range correlated vortices. Notice that the two-dimensional energy spectrum $E(q) = q\langle \vec{u}(\vec{q})\cdot\vec{u}(-\vec{q})\rangle/(4\pi V)$, can be reexpressed with the charge structure factor as $E(q)=n/(4\pi)q\; v^2(q)S_{ZZ}(q)$. The interpretation of the classical two-dimensional turbulence spectral laws $E(k)\sim k^{-\mu}$ ($\mu=3$ [20,45,46], $\mu \simeq 4$ [11,47]), in terms of quasi-point-like vortices $[v(q)\sim q^{-1}]$ leads to $S_{ZZ}\sim q^{-(\mu-1)}$. This is supposed to be the behavior of S_{ZZ} in the inertial range. When the structure factor is steep enough (say, if μ is significantly larger than 1), one expects the vorticity correlations to be long range and positive: the spatial structure differs qualitatively from that of the ideal gas given by a white spectrum. The common spectral laws are compatible with preferential ordering between same sign vortices.

At large scales, the flow is made of large coherent vortical structures. The pair correlations between these vortices are closely related to the properties of the energy spectrum in the low wave number limit, outside of the inertial range. For instance, if the spectrum in this limit is such that $E(q) \sim q^{\mu'}$, with $\mu'>0$ [48], the electroneutrality is local beyond the size of the coherent vortices. In that case, one expects that the structure of the vortex system should be similar to the short range structure in liquids.

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APPENDIX

For identical vortices and with the help of relation (2.5), the diagonal terms $(a)=(b)$ of the sum

$$\frac{1}{V}[\hat{k}_0\cdot\vec{u}(\vec{q})][\hat{k}_0\cdot\vec{u}(-\vec{q})]$$
$$=\frac{1}{V}\sum_{a,b}\langle[\hat{k}_0\cdot\vec{v}^{(a)}(\vec{q})][\hat{k}_0\cdot\vec{v}^{(b)}(-\vec{q})]\rangle \quad \text{(A1)}$$

give the contribution

$$n\int d\hat{a}[\hat{k}_0\cdot\vec{v}(\vec{q},\hat{a})][\hat{k}_0\cdot\vec{v}(-\vec{q},\hat{a})], \quad \text{(A2)}$$

where $\vec{v}(\vec{q},\hat{a})=\int d\vec{x}\vec{v}(\vec{x},\hat{a})\exp(-i\vec{q}\cdot\vec{x})$. The off-diagonal terms $(a)\neq(b)$ take the form

$$\frac{N^2}{V}\frac{1}{V^2}\int d\vec{x}d\vec{x}'d\vec{x}_a d\vec{x}_b d\hat{a}d\hat{b}g(\vec{x}_a-\vec{x}_b,\hat{a},\hat{b})$$
$$\times e^{-i\vec{q}\cdot(\vec{x}_a-\vec{x}_b)}\hat{k}_0\cdot\vec{v}(\vec{x},\hat{a})e^{-i\vec{q}\cdot\vec{x}}\hat{k}_0\cdot\vec{v}(\vec{x}',\hat{b})e^{i\vec{q}\cdot\vec{x}'} \quad \text{(A3)}$$

and can be recast as

$$n^2\int d\hat{a}d\hat{b}[\hat{k}_0\cdot\vec{v}(\vec{q},\hat{a})][\hat{k}_0\cdot\vec{v}(-\vec{q},\hat{b})]h(\vec{q},\hat{a},\hat{b}). \quad \text{(A4)}$$

Summing the terms (A2) and (A4), the relations (2.2) and (2.4) are transformed to Eqs. (2.7) and (2.8).

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[43] As noted in the Introduction, two-dimensional turbulence has recently become the object of renewed experimental interest [20,21], and the latter two references may point out directions for the experimental verifications of the results and ideas put forth in the present paper. Also, given the well known isomorphism between acoustic waves in a slightly compressible vortical fluid and surface waves over a vortical liquid, a further experimental possibility comes to mind: study the interaction between surface waves and vertical vorticity in water tanks; see F. Vivanco, F. Melo, C. Coste, and F. Lund, Phys. Rev. Lett. **83**, 1966 (1999).

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